A MAGNETOHYDRODYNAMIC POWER GENERATOR SYSTEM FOR SUBMARINE APPLICATION

by

Lieutenant Stephen Michael Pattin, U.S. Navy

B.S. United States Naval Academy 1956

Submitted in Partial Fulfillment
of the Requirements for the Degree of

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Submitted to the Department of Naval Architecture and Marine Engineering on May 17, 1963, in partial fulfillment of the requirements for the Master of Science degree in Naval Architecture and Marine Engineering and the Professorial degree, Naval Engineer.

ABSTRACT

This thesis presents an analysis of a direct current conduction magnetohydrodynamic generator. Laminar, incompressible viscous flow is considered. Experimental evidence is offered to show that the laminar assumption is valid in the presence of strong magnetic fields.

Efficiencies are calculated for generators of various length and aspect ratio, efficiency curves are presented and a method of driving the flow is considered.

Thesis Supervisor: Herbert H. Woodson
Title: Professor of Electrical Engineering

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Chapter I

INTRODUCTION

1.1 Background

The minimization of sound transmitted from a submarine through the sea to the listening devices of a potential enemy is a major problem in submarine design. A large fraction of the radiated noise is generated by the submarine's propulsion plant: the main engines and the screws and shafting

Consequently it would by desirable to devise a propulsion system having neither shaft nor screw and a minimum quantity of rotating machinery. Doragh* (1) has shown the feasibility of propelling a submarine by means of a sea water jet accelerated in a magnetohydrodynamic channel. This thesis will be a study of the closely related problem of providing the electrical energy necessary to drive the magnetohydrodynamic flow.

The pioneer figure in the study of magnetohydrodynamic channel flow was Hartmann (2). In 1937 he made the first detailed theoretical analysis of the flow of a conducting fluid in a high aspect ratio channel under the influence of a uniform transverse magnetic field. Hartmann and Lazarus (3), in that same year, reported on observations made on mercury flow in a transverse magnetic field. To date the principal use of magnetohydrodynamic channel flow has been as electromagnetic pumps to circulate liquid metal coolants in atomic power installations.

At the present time there is considerable interest in the development of magnetohydrodynamic power generators.

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* The bracketed numbers refer to the Bibliography which begins on Page 28

Most of these proposed generators make use of a conducting gas or plasma as the working fluid. This type of magnetohydrodynamic generating system shows promise of supplying large quantities of electric power at thermal efficiencies unattainable with present equipment (4,5,6,7).

1.2 The Object and Scope of This Study

The object of this study is the design of a liquid metal magnetohydrodynamic direct current conduction generator compatible with and utilizing the same magnetic field as the magnetohydrodynamic pump designed by Doragh. As will be shown, the use of a liquid metal, instead of a conducting gas or plasma, as the working fluid permits the design of a complete energy conversion cycle devoid of mechanical pumps and compressors. Since the basic goal of this design is to achieve a system free of these mechanical noise sources, gaseous working fluids will not be considered.

The channel under consideration is rectangular in cross section and of high aspect ratio. The applied transverse magnetic field is uniform and parallel to the shorter sides of the channel cross section. Furthermore, the liquid metal is assumed to be isotropic, homogeneous and incompressible.

The study will be restricted to laminar viscous flow, since there is little or no experimental data available concerning turbulent flow of an electrically conducting fluid in a strong transverse magnetic field. In fact, the assumption of laminar flow may be quite valid since there is some experimental evidence that indicates a transverse magnetic field delays the transition from laminar to turbulent flow. Murgatroyd (8) in his experiments with mercury found that laminar flow was stable when the hydraulic Reynolds number was less than 225 times the Hartmann number. Lock (9) hypothesized that laminar flow would be stable for a Reynolds number less than 42,000 times the Hartmann number

for larger Hartmann numbers. Since this study will be dealing only with large Hartmann numbers, both theory and experiment predict laminar flow.

Chapter II
PROCEDURE

2.1 Basic Equations

The fluid under consideration is homogeneous, isotropic, non-magnetizable, non-polarizable and incompressible. It is characterized by a permeability μ_o, an electrical conductivity a mass density ρ and a dynamic viscosity η . The equations that govern the motion of such a fluid are the magnetohydrodynamic Navier-Stokes equation.

$$\rho\frac{D\bar{v}}{Dt} = -\text{grad } p + \nabla^2 \bar{v} + \bar{J} \times \bar{B}, \qquad (2.1.1)$$

the incompressibility condition

$$\text{div } \bar{v} = o, \qquad (2.1.2)$$

Maxwell's equations

$$\text{curl } \bar{H} = \bar{J}, \qquad (2.1.3)$$
$$\text{curl } E = -\mu_o\frac{\partial \bar{H}}{\partial t}, \qquad (2.1.4)$$

and

$$\text{div } \bar{H} = o; \qquad (2.1.5)$$

and the constitutive relation

$$\bar{J} = \sigma(\bar{E} + \bar{v} \times \mu_o\bar{H}). \qquad (2.1.6)$$

Where $\bar{H}$ is the magnetic field intensity, $\bar{J}$ is the conduction current density, $\bar{E}$ is the electric field intensity, $\bar{v}$ is the fluid velocity and P is the pressure. Rationalized MKS units are used in all equations and all equations are written with respect to a fixed coordinate system.

2.2 Pressure and Velocity Relations

The geometry of the generator to be considered is illustrated in Figure I. Fluid flows in the z-direction with a velocity that is a function of both x and y. A uniform magnetic field $\bar{H}$ is applied in the x direction. The channel walls at $y = \pm w/2$ are highly conducting buses that are maintained at a potential difference by the external constraint imposed by the load resistance. The walls at $x = \pm d/2$ are non-conducting. The electric field $\bar{E}$ resulting from the transverse potential gradient is assumed to be in the negative y direction, and the current density $\bar{J}$ is assumed to be in the y direction.

The channel cross section is shown in Figure II. Load current I is assumed to flow symmetrically in the external circuit. This insures that the magnetic field resulting from the flow of load current will be in the z direction and so will not affect the generator operation. End effects will not be considered at this time.

Under these assumptions, the z component of Equation (2.1.1) becomes

$$0 = -\frac{\partial p}{\partial z} + \eta\frac{\partial^2 v_z}{\partial x^2} + \eta\frac{\partial^2 v_z}{\partial y^2} - J_y B_x \qquad (2.2.1)$$

The elimination of J_y by use of Equation (2.1.6) yields

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - \frac{\sigma}{\eta} B^2 v + \frac{\sigma}{\eta}\left(EB - \frac{1}{\sigma}\frac{\partial p}{\partial z}\right) = 0 \, * \qquad (2.2.2)$$

This equation can be written as

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* From this point on indices indicating vector directions will be omitted. All directions are as defined in Figure I.

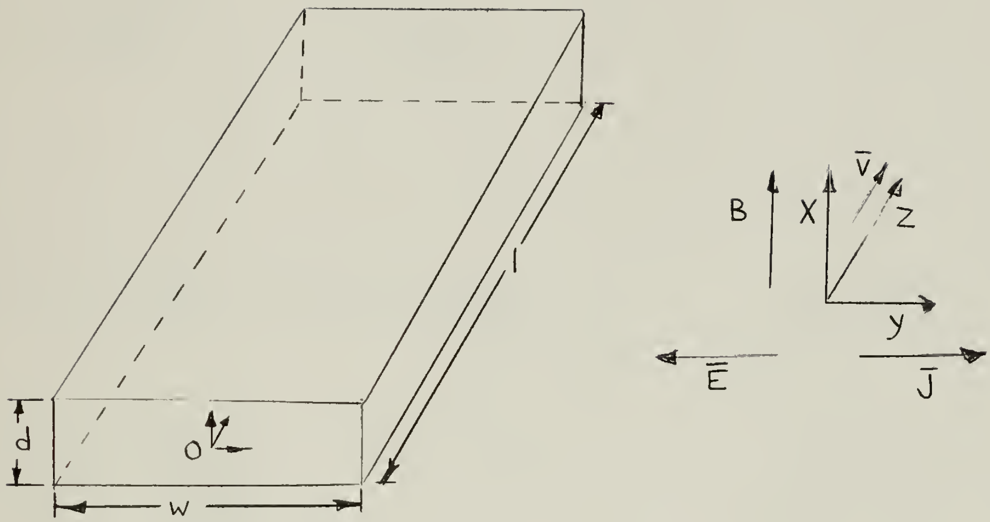

Figure I
Channel Geometry

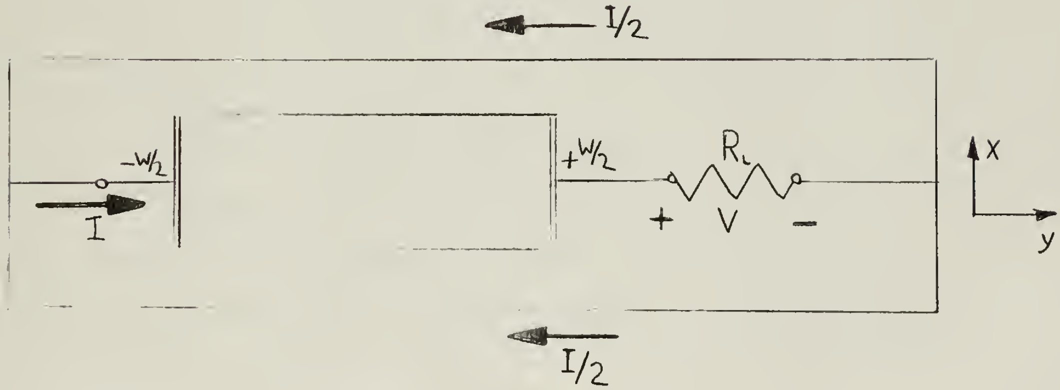

Figure II
Channel Cross Section and External Electric Circuit

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - k^2 v + P = 0 \qquad (2.2.3)$$

where

$$k^2 = \frac{B^2 \ (d/2)^2}{-(d/2)^2} = \frac{M^2}{(d/2)^2} \qquad (2.2.4)$$

,

$$M = B(d/2)\sqrt{\frac{\sigma}{\eta}} \text{ (Hartmann number)} \qquad (2.2.5)$$

and

$$P = \frac{\sigma}{\eta}(EB - \frac{1}{\sigma}\frac{\partial p}{\partial z}) \qquad (2.2.6)$$

A solution to Equation (2.2.3) of the form

$$v = \sum_{n=1}^{\infty} v_{2n-1}(x) \cos \frac{(2n - 1)\pi y}{w} \, dy \qquad (2.2.7)$$

is desired, where

$$v_{2n-1}(x) = \frac{2}{w}\int_{-w/2}^{w/2} v \cos \frac{(2n-1)\pi y}{w} \, dy \qquad (2.2.8)$$

If Equation (2.2.3) is multiplied by the function $\frac{2}{w} \cos \frac{(2n-1)\pi y}{w} \, dy$ and the resulting equation integrated over the interval $y = -w/2$ to $y = w/2$, and the boundary condition, $v(x, \pm w/2) = 0$ is applied, the result is

$$\frac{\partial^2 v_{2n-1}}{\partial x^2} - \left[\frac{M^2}{(d/2)^2} + \left[\frac{(2n-1)\pi}{w}\right]^2 \right. v_{2n-1} + \frac{4P(-1)^{n+1}}{(2n-1)\pi} \mp 0 \qquad (2.2.9)$$

or

$$\frac{\partial^2 v_{2n-1}}{\partial x^2} - M_n^2 \, v_{2n-1} + \frac{4P(-1)^{n+1}}{(2n-1)\pi} = 0 \qquad (2.2.10)$$

where

$$M_n^{\ 2} = \frac{M^2}{(d/2)}\,^2 + \frac{(2n-1)\pi}{w} \qquad (2.2.11)$$

Define the function

$$\delta_{2n-1} = M_n^{\ 2} v_{2n-1} + \frac{4P(-1)^{n+1}}{(2n-1)\pi} \qquad (2.2.12)$$

then Equation (2.2.10) can be expressed as

$$\frac{\partial^2 \delta_{2n-1}}{\partial x^2} - Mn^2 \delta_{2n-1} = 0 \qquad (2.2.13)$$

This has the solution

$$\delta_{2n-1} = A\cosh M_n x + B \sinh M_n x \qquad (2.2.14)$$

where A and B are arbitrary constants to be evaluated from the boundary conditions.

Because of the symmetry of the problem, v, and hence δ_{2n-1} must be an even function of x. Therefore B = 0. A may be evaluated from the requirement that $v(\pm d/2,y) = 0$.

The solution for δ_{2n-1} is then

$$\delta_{2n-1} = \frac{-4P(-1)^{n+1}}{(2n-1)\pi \cosh \frac{M_n d}{2}} \qquad (2.2.15)$$

and

$$v_{2n-1} = \frac{4P(-1)^{n+1}}{(2n-1)\pi M_n^{\ 2}} \quad 1 - \frac{\cosh M_n x}{\cosh \frac{M_n d}{2}} \qquad (2.2.16)$$

whence

$$v = \frac{4P}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)M_n^{\ 2}} \left[1 - \frac{\cosh M_n x}{\cosh \frac{M_n d}{2}} \right] \cos \frac{(2n-1) 1 \pi y}{w} \qquad (2.2.17)$$

The expression for the mean velocity $\underline{v}$ can be found by integrating Equation (2.2.17) across the channel with the result

$$\underline{v} = \frac{8P}{\pi^2} \sum_{m=1}^{\infty} \frac{1}{(2n-1)^2 M_n^{\ 2}} \left[1 - \frac{2}{M_n d} \tanh \left(M_n \frac{d}{2}\right) \right]. \qquad (2.2.18)$$

Equations (2.2.6) and (2.2.18) can now be combined to give the pressure gradient down the channel.

$$P_o = \frac{\partial p}{\partial z} = \sigma EB - \frac{\pi^2 \underline{v}n}{8\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2 M_n^{\,2}} \left[1 - \frac{2}{M_n d}\tanh\left(M_n \frac{d}{2}\right)\right]} \quad *$$

(2.2.19)

2.3 Terminal Impedance

For large Hartmann numbers Equation ω2.2.17) predicts that the normal parabolic velicoty profile associated with laminar flow is distorted by the presence of the transverse magnetic field. The flow in the center of the channel is retarded and that near the walls accelerated so that the result is a core of uniform flow and a boundary layer whose thickness is of the ordere of d/M. **

In determining the terminal characteristics of the generator the velocity will be assumed to be uniform and equal to $\underline{v}$ The Hartmann number will be large enough to make this assumption valid.

With the assumption of a uniform velocity profile, the equivalent electrical circuit for the generator can be determined quite easily from Equation (2.1.6). After transforming this equation from field variables to circuit variables the result is

$$Ild \,.\, \left(\underline{v}B + \frac{V}{w}\right). \qquad (2.3.1)$$

I and V are positive in the directions defined in Figure II.

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* For a complete discussion of these results see Ryabinin and Khozhainov (10).

** See Shercliff (11) page 137.

This current-voltage relationship determines the equivalent electrical circuit shown schematically in Figure III.

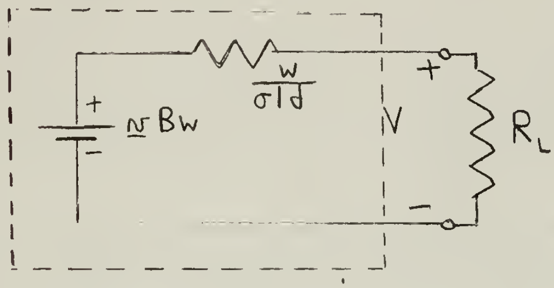

Figure III

A characteristic of this generator is its low terminal voltage. This is readily seen from the expression for open circuit voltage, $\underline{v}Bw$. For a liquid metal the maximum mean velocity practiable is of the order of 10 meters per second. The channel width is of the order of one meter. The $\bar{B}$ field used by Doragh, and hence the field acting on this generator is ten Webers per square meter. The open circuit voltage then is limited to a maximum of about 100 volts.

2.4 Power Relations

Define R_i, the internal resistance of the generator, by

$$R_i = \frac{w}{\sigma l d}. \tag{2.4.1}$$

The terminal voltage, V, can then be expressed as

$$V = \frac{\underline{v}BwR_L}{R_i + R_L}, \tag{2.4.2}$$

and the load current, I, as

$$I = \frac{\underline{v}Bw}{R_i + R_L}. \tag{2.4.3}$$

The electric power delivered to the load, P_L, is given by

$$P_L = I^2 R_L = \frac{(\underline{v}Bw)^2 R_L}{(R_i + RL)^2} \quad . \tag{2.4.4}$$

And the generator efficiency, e, is given by

$$e = \frac{P_L}{\text{Rate of doing mechanical work on the fluid}}$$

$$e = \frac{P_L}{P_o \underline{v}\ lwd} = \frac{(\underline{v}Bw)^2 R_L}{(R_i + R_L)^2 P_o \underline{v}\ lwd} \tag{2 4.5}$$

2.5 The Energy Conversion Cycle

The source of energy for operating the generator is assumed to be a nuclear reactor. The problem of converting the thermal energy of thw reactor into mechanical energy necessary to drive the generator without the intervention of mechanical pumps, compressors and other mechanical noise sources is a difficult one. The cycle considered here was devised by Elliot (12). It is shown schematically in Figure IV.

Elliot carries out the thermal to mechanical energy conversion by mixing a hot liquid metal of low vapor pressure with a cooler liquid metal of higher vapor pressure. The temperature of the hot metal is above the boiling point of the cooler metal, and in the mixing process the lower temperature metal vaporizes. The mixture of vapor and liquid metal droplets is passed into a nozzle where the vapor expands and accelerates the liquid phase. The vapor and liquid phases are then separated from each other. The vapor is condensed and pumped back to the mixing chamber by means of a magnetohydrodynamic

pump. The liquid flows through the generator, to the heat source and back to the mixer.

The essential feature of this cycle is that it has no moving parts and should satisfy the requirement for noise-free operation to a high degree. Its major drawbacks are that it is relatively inefficient and the problem of complete separation of the two phases has not yet been solved.

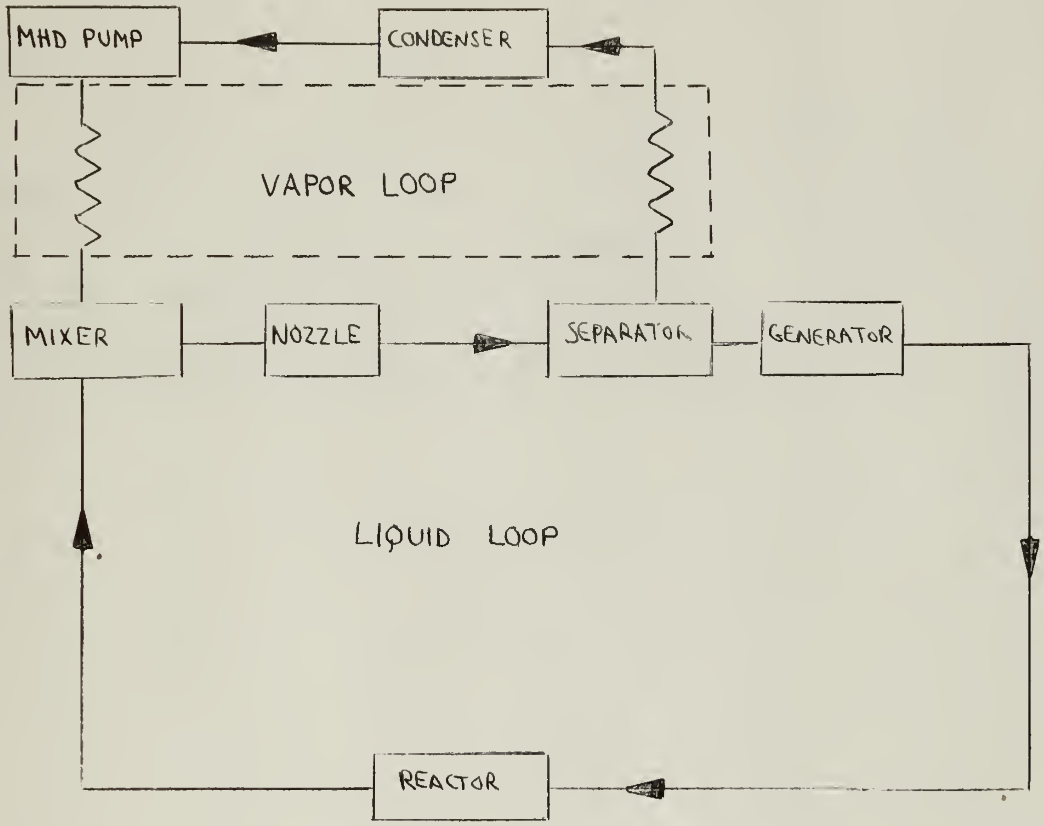

Figure IV A Proposed Power Generation Cycle

Chapter III

RESULTS

3 1 Results

Generator efficiencies were calculated for generators of several lengths and aspect ratios for different mean flow velocities. The resultant plots of efficiency versus velocity are shown in Figures V, VI and VII. Figures VII, IX and X show characteristics of a generator one meter wide, one meter long and with an aspect ratio of five.

Liquidlithium was assumed to be the working fluid in all cases.

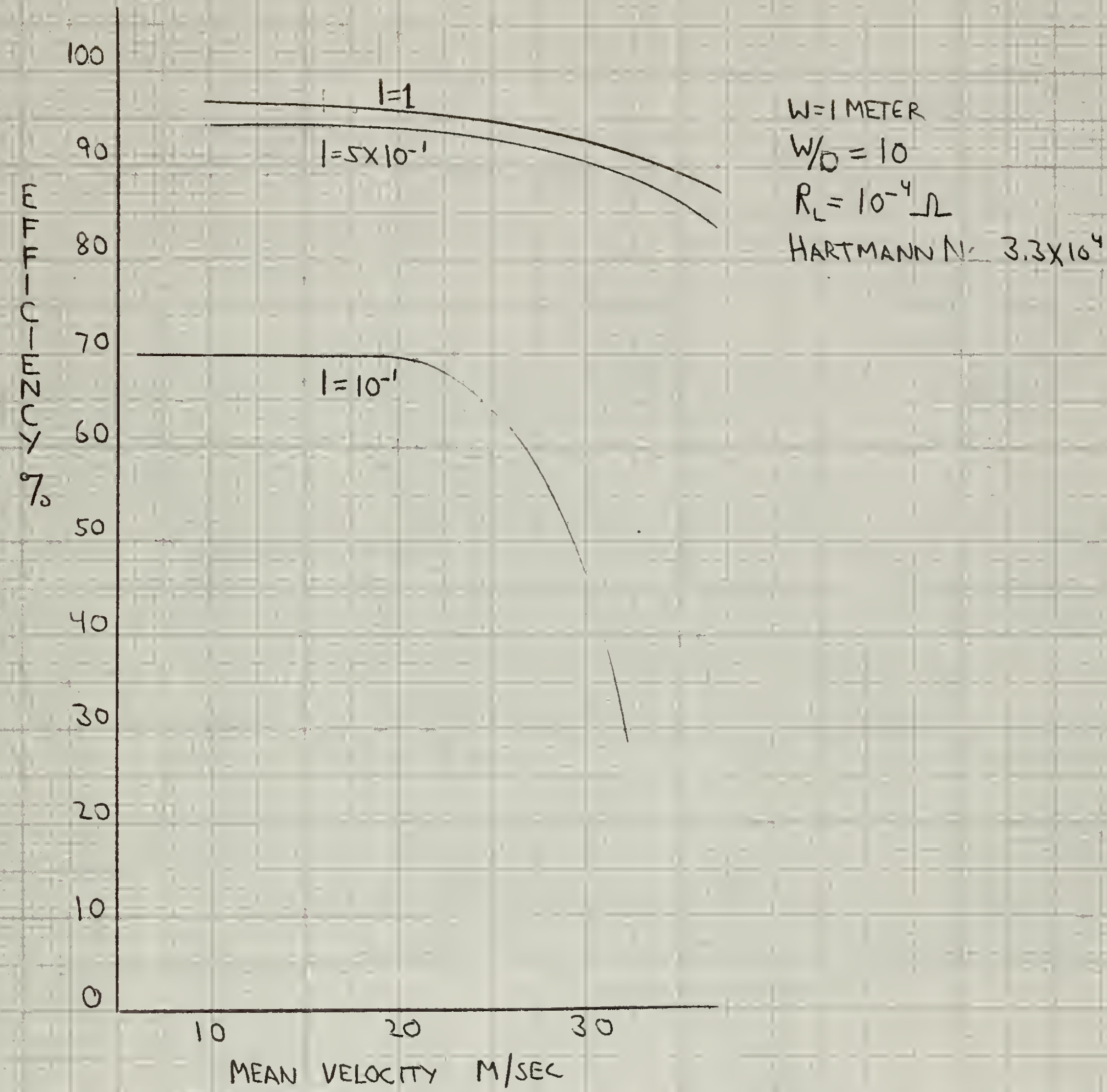

Figure V Efficiency vs. Velocity for 10/1 Generator

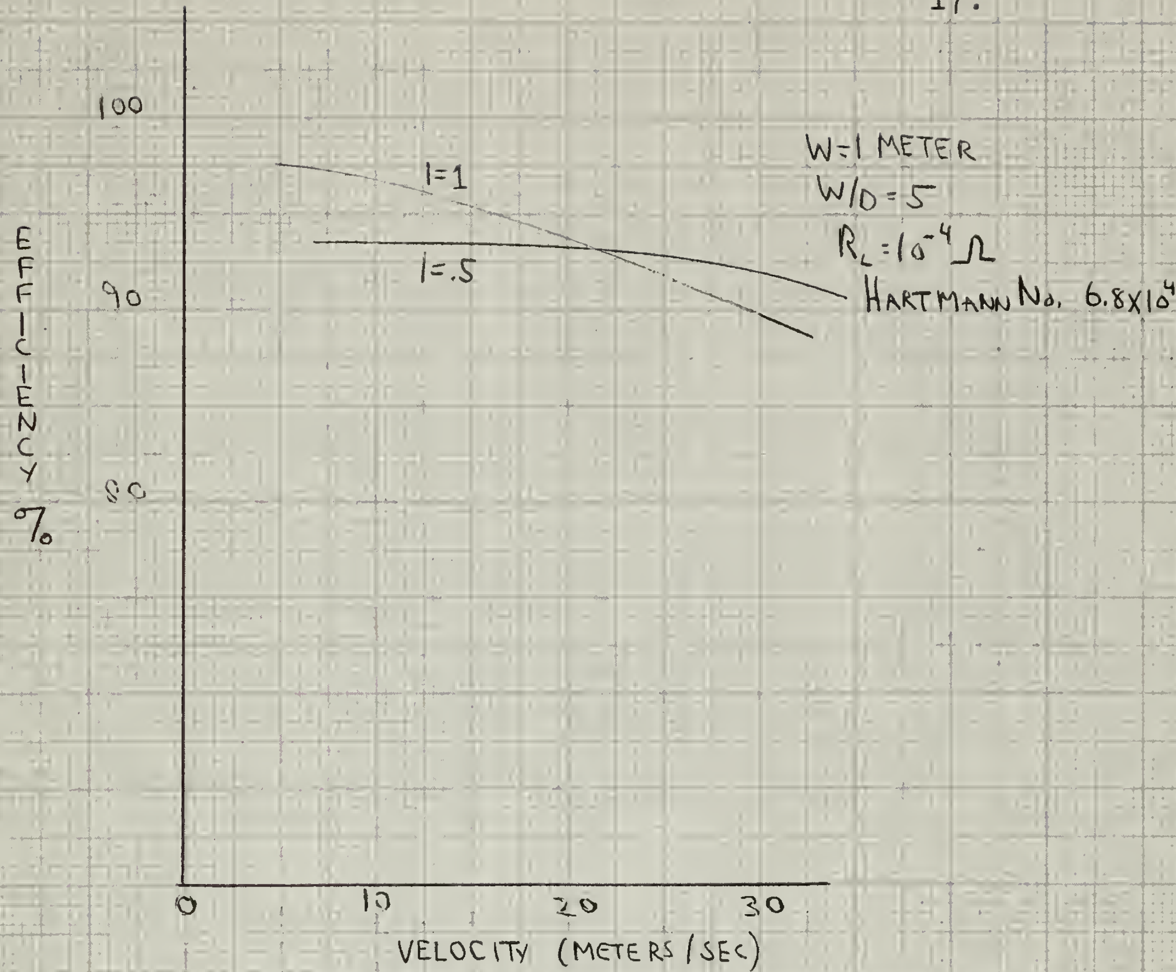

Figure VI Efficiency vs. Velocity for 5/1 Generator

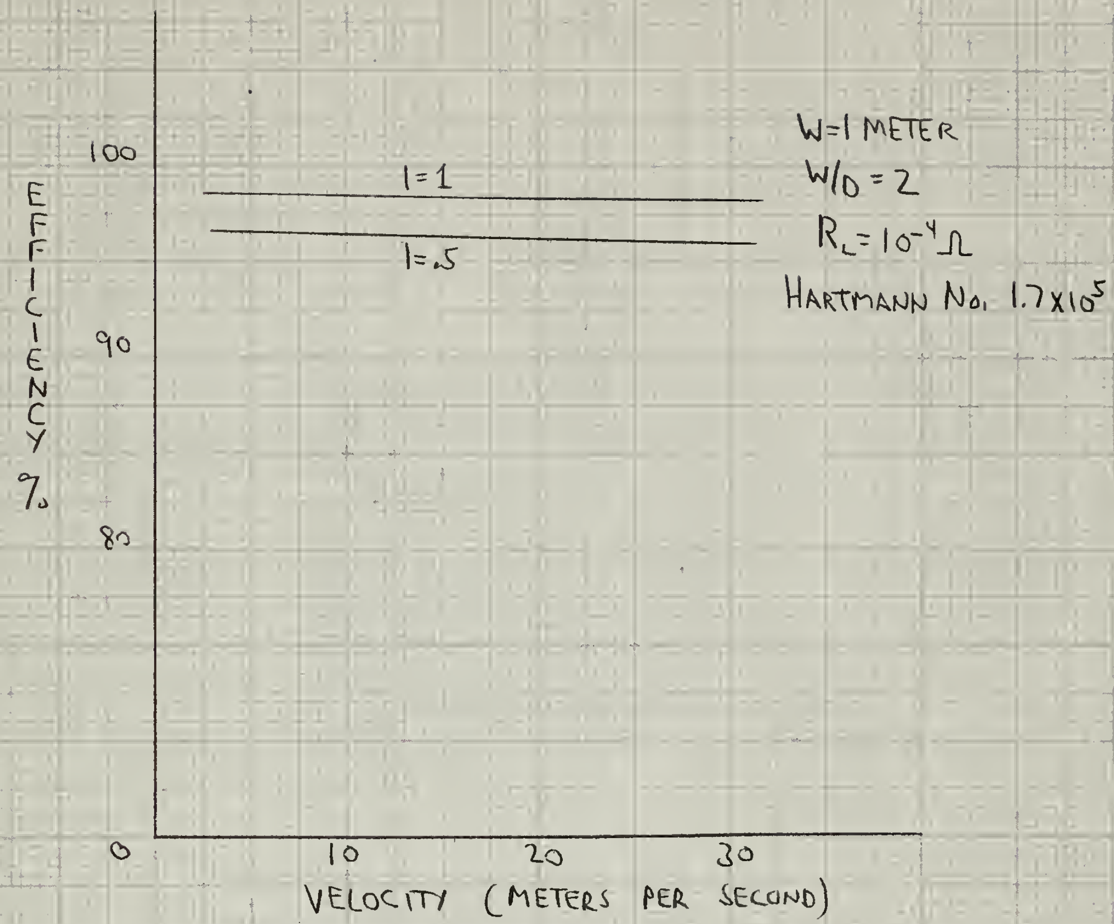

Figure VII Efficiency vs. Velocity for 2/1 Generator

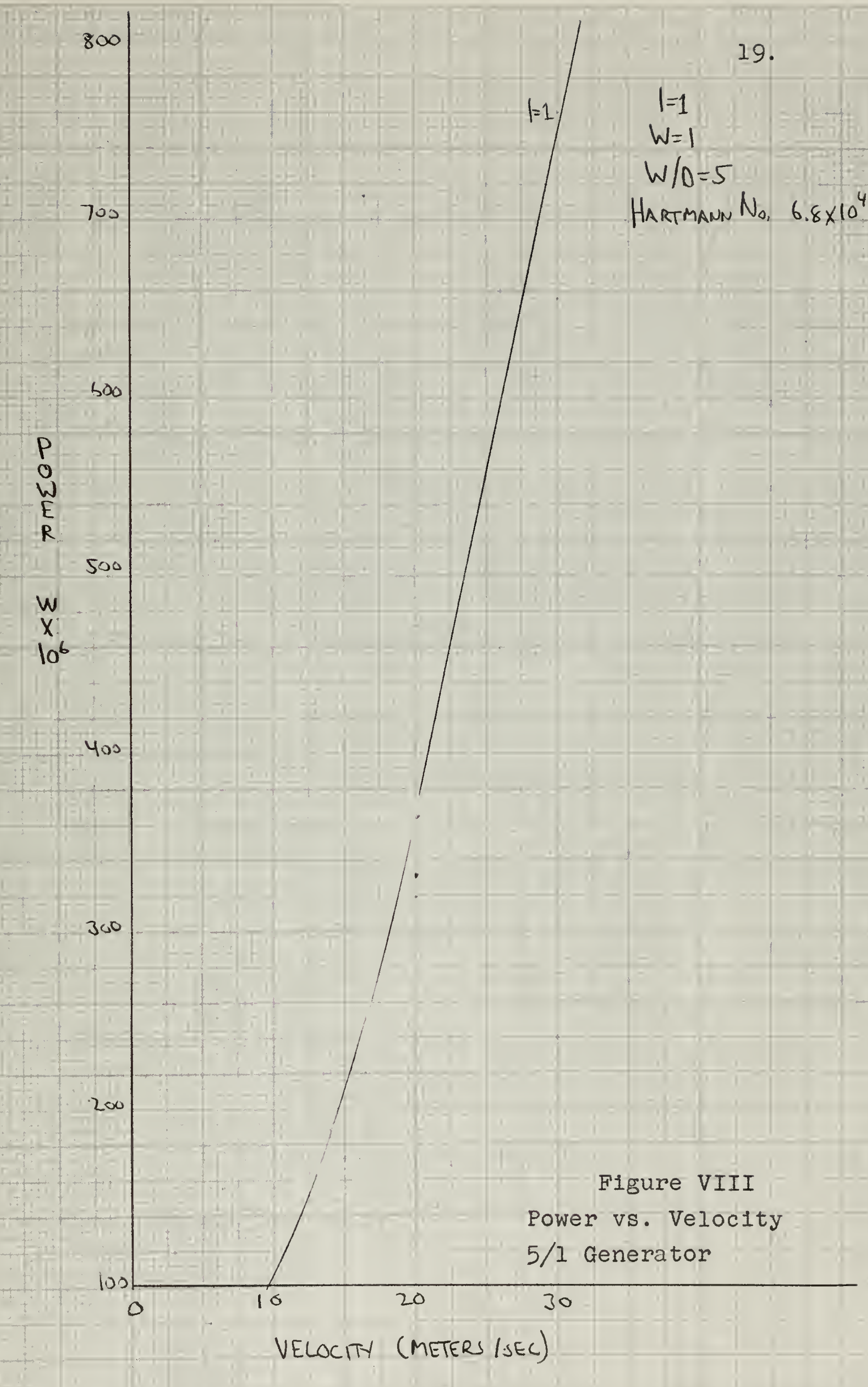

Figure VIII
Power vs. Velocity
5/1 Generator

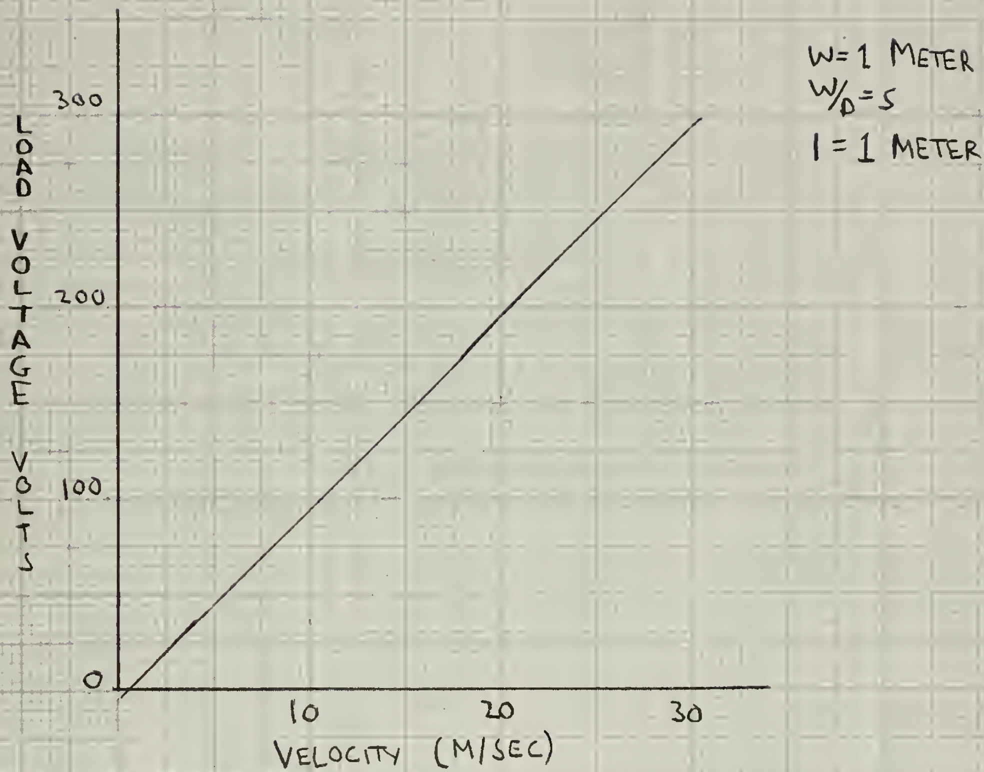

Figure IX Voltage vs. Velocity

5/1 Generator

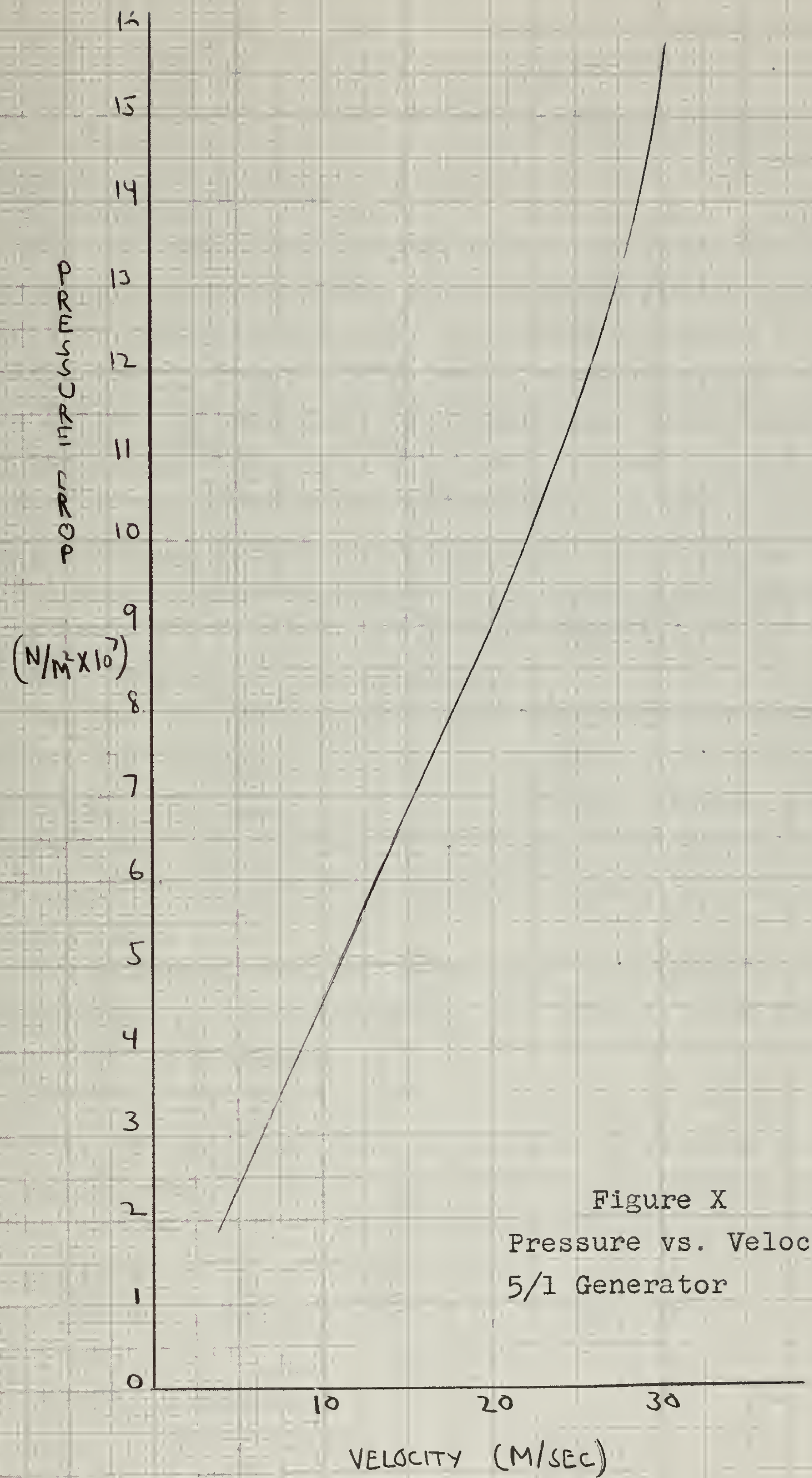

Figure X
Pressure vs. Velocity
5/1 Generator

Chapter IV

DISCUSSION OF RESULTS

4.1 Discussion

The results show that, with the exception of the thin high aspect ratio channel there was relatively little change in efficiency as the velocity was varied throughout the useful range. Also, the longer channels were more efficient than the short channels. This result was to be expected since the very low load impedence coupled to the generator caused large current flows. These currents were large enough that the viscous losses in the fluid were insignificant when compared with ohmic losses. Viscous forces were important only in the thin channel at high speeds.

The results also demonstrate the obvious fact that channel efficiency increases as channel cross section is increased Viscous losses are basically surface effects and anything that is done to decrease the ratio of surface area to volume of the channel serves to lower the viscous losses.

Ohmic losses, on the other hand, are increased as channel width is increased and are lowered when length or height is increased.

At this time it might be worthwhile to test the validity of the assumption of laminar flow upon which the whole analysis was based For liquid lithium* in a magnetic fluz density of ten webers per square meter the Hartmann numbers ranged from 6×10^4 to 2×10^5 The hydraulic Reynolds number for these same flows ranged from 10^6 to 10^7 The ratio of Reynolds number to Hartmann number was at all times less than the critical value of 225 found by Murgatroyd. There is

* For the properties of lithium see Appendix A

[illegible]

still some question as to the validity of the relationship for the critical Reynolds number being equal to 225 times the Hartmann number for the extremely high Hartmann numbers encountered here. In this respect it is interesting to note that, in his experiments, Murgatroyd was unable to obtain turbulent flow for a Reynolds number of 10^5.

One of the major disadvantages of the constant velocity channel is the large pressure gradient required to produce any significant amount of electric energy. Since the liquid metal is incompressible there is no conversion of thermal energy to electrical energy within the channel. The kinetic energy of the flow also is constant in the channel Thus the difference between generator inlet pressure and outlet pressure alone must account for both channel losses and power delivered to the load.

Another disadvantage of the liquid metal channel as a generator is the low terminal voltage available. It is desirable to operate at low flow velocities to keep viscous losses low. Since the terminal voltage of the device is dictated by the product of magnetic flux density, velocity and channel width, a terminal voltage of no more than one hundred volts is expected. This low voltage requires a low load impedence in order to extract any significant amount of electrical energy from the flow.

4.2 Conclusion

The liquid metal conduction generator is extremely attractive because of its small size and simplicity. The major disadvantages listed above are serious but not in themselves sufficient grounds for discarding the device.

It is only when one looks further than the generator itself that insurmountable difficulties appear. The absence of an efficient method for converting thermal energy to flow energy without using mechanical equipment is a severe handicap. Elliot's cycle mentioned

earlier has many drawbacks. It is inherently inefficient and its temperature requirements exceed the capabilities of presently available materials. The cycle requires the use of a liquid metal with a high boiling point in the liquid loop. It follows that any metal meeting this requirement would be solid at room temperature. As a result a means must be provided for keeping the metal from freezing when the system is not in use. If lithium is used this means maintaining a temperature of above 354° F.

The flow could be driven by a mechanical pump operating in a conventional steam cycle. However, this would defeat the major reason for having an MHD generator: noise elimination. Besides, if a conventional steam cycle must be built to operate a pump, it may just as well drive a rotating generator directly.

Chapter V

CONCLUSIONS AND RECOMMENDATIONS

5.1 Advantages

The prime advantage of the generator studied is its small size and simplicity. In addition, it meets the basic requirement of relatively noise-free operation.

5.2 Disadvantages

The disadvantages of this type of generator system are manifold. Terminal voltage is low, losses in the system excluding the generator are relatively high and there is a serious materials problem.

5.3 Recommendations

Since the difficulties of producing electrical energy by means of a liquid metal cycle far outweigh the advantages of such a system, as study of an MHD power generator system using an ionized gas as a working fluid would certainly be in order. If such a system could be made small enough in physical size to be placed in a submarine it might very well provide the quiet operation we are seeking.

APPENDIX A

THE PROPERTIES OF LIQUID LITHIUM (13)

Melting Point		354° F.	
Boiling Point		2403° F.	
Density		.507 gm/cc	200° C.
		.490	400
		.474	600
		.457	800
		.441	1000
Viscosity	.5918	Centerpoises	183.9° C.
	.5749		193.2
	.5541		208.1
	4917		250.8
	.4548		205.5
Resistivity	45.25	micro ohm-cm	230° C.

Hartmann Number $= B\left(\frac{d}{2}\right)\sqrt{\frac{\sigma}{\eta}} = 6.82 \times 10^5\left(\frac{d}{2}\right)$

APPENDIX B

METHODS OF COMPUTATION

All calculations were made using the properties of lithium listed in Appendix A

Equation (2.2.19) was solved for P_o, Equation (2.2.4) for P_L and Equation (2.4.5) then solved to determine efficiency.

A magnetic flux density of ten webers per square meter was assumed in all calculations.

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